Musterlösung Aufgabe 1: ≪Druckgasbehälter≫

I. TEILAUFGABE A) $\Rightarrow$ 2 PUNKTE

Mit der Isentropenexponente $\kappa = \frac{c_p}{c_v} \quad \Rightarrow p \cdot V^\kappa = const$

$c_v = c_p - R = c_p - \frac{R_m}{M}$ $c_v = 2.22 \frac{kJ}{kg \cdot K} - \frac{8.3145 \frac{kJ}{kmol \cdot K}}{16 \frac{kg}{kmol}} = 1.7 \ \frac{kJ}{kg \cdot K}$

Isentropenexponent eines idealen Gases $\kappa = \frac{c_p}{c_v}$

$\kappa = \frac{2.22 \frac{kJ}{kg \cdot K}}{1.7 \frac{kJ}{kg \cdot K}} = 1.306 \ \frac{kJ}{kg \cdot K}$

II. TEILAUFGABE B) $\Rightarrow$ 3 PUNKTE

Isentrope Kompression eines idealen Gases: $p \cdot v^\kappa = const.$

$p_1 \cdot v_1^\kappa = p_2 \cdot v_2^\kappa.$

mit $p \cdot v = R \cdot T \quad \Rightarrow \quad \frac{T_2}{T_1} = (\frac{p_2}{p_1})^{\frac{\kappa-1}{\kappa}}$

$T_2 = T_1 \cdot (\frac{p_2}{p_1})^{\frac{\kappa-1}{\kappa}}$

$T_2 = (273.15 + 10)K \cdot (\frac{100}{5})^{\frac{1.306-1}{1.306}} = 571.28 \ K = 298.13°C$

III. TEILAUFGABE C) $\Rightarrow$ 2 PUNKTE

Gasmasse

$p \cdot V = m \frac{R_m}{M} T \quad \Rightarrow \quad p_2 \cdot V_2 = m_2 \frac{R_m}{M} T_2$

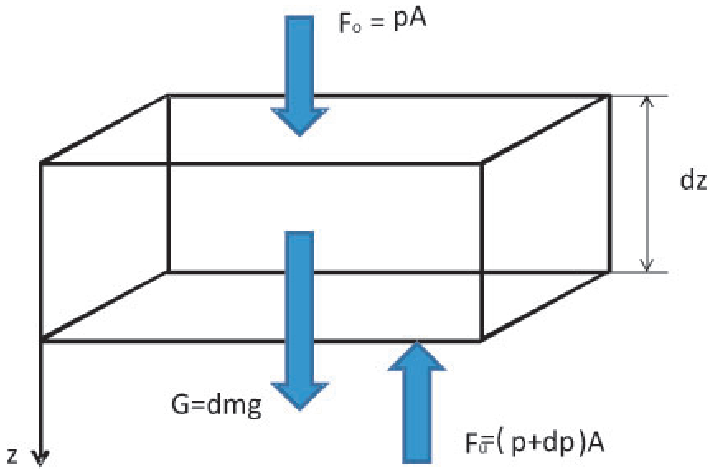
$m_2 = \frac{p_2 V_2 M}{R_m T_2} \quad \Rightarrow \quad m_2 = \frac{100 \cdot 10^5 (\frac{N}{m^2}) \cdot 300000(m^3) \cdot 16(\frac{kg}{kmol})}{8.3145(\frac{N \cdot m}{mol \cdot K}) \cdot 571.28(K)} = 1.01 \cdot 10^7 (kg)$

IV. TEILAUFGABE D) $\Rightarrow$ 3 PUNKTE

spezifische Arbeit für die Kompression: $q_{12} + w_{t12} = h_2 - h_1 = c_p(t_2 - t_1)$

$q_{12} = 0 \quad \Rightarrow \quad w_{t12} = c_p(t_2 - t_1) = 2.22(\frac{kJ}{kg \cdot K})(298.13 - 10)K = 639.65 \ (\frac{kJ}{kg})$

V. TEILAUFGABE E) $\Rightarrow$ 4 PUNKTE



$$G = dm \cdot g = \rho(z) \cdot dV \cdot g = [dV = A \cdot dz] = \rho \cdot dz \cdot A \cdot g$$

$$F_u = (p + dp)A$$

$$F_o = pdA$$

$$\sum F_i = 0 \quad \Rightarrow G + F_o - F_u = 0$$

$$\rho \cdot dz \cdot A \cdot g - (p + dp)A + pA = 0 \quad \Rightarrow \rho \cdot dz \cdot g + dp = 0 \quad \Rightarrow \frac{dp}{dz} = \rho \cdot g$$

$$\text{mit} \quad \rho = \frac{p \cdot M}{R \cdot T} \quad \Rightarrow \frac{dp}{dz} = \frac{p \cdot M \cdot g}{R \cdot T(z)} \quad \Rightarrow \frac{dp}{p} = \frac{dz \cdot M \cdot g}{R \cdot T(z)}$$

$$\int \frac{dp}{p} = \int \frac{M \cdot g}{R \cdot T(z)} dz$$

$$\text{mit} T(z) = T_E + 30 \cdot z(\tfrac{K}{km}) \quad \Rightarrow \quad \int \frac{dp}{p} = \frac{M \cdot g}{R} \int \frac{dz}{T_E + 30 \cdot z}$$

$$\text{mit} \quad \int \frac{dx}{ax + b} = \frac{1}{a} \ln(ax + b) \quad \Rightarrow \quad \ln \frac{p_z}{p_0} = \frac{M \cdot g}{R} |_0^z \langle \frac{1}{30} \ln(T_E + 30 \cdot z) \rangle$$

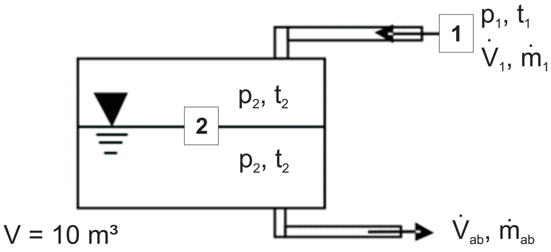
$$\ln \frac{p_z}{p_0} = \frac{M \cdot g}{R \cdot 30} \langle \ln(T_E + 30 \cdot z) - ln(T_E + 30 \cdot 0) \rangle$$

$$\ln \frac{p_z}{p_0} = \frac{M \cdot g}{R \cdot 30} \langle \ln(1 + \tfrac{30 \cdot z}{T_E}) \rangle$$

$$p(z) = p_0 \left[1 + \frac{30 \cdot z}{T_E}\right]^{\dfrac{M \cdot g}{R \cdot 30}} \quad \Rightarrow \quad p_0 = p(z) / \left[1 + \frac{30 \cdot z}{T_E}\right]^{\dfrac{M \cdot g}{R \cdot 30}}$$

$$p_0 = 100(bar) / \left[1 + \frac{30 \cdot 2(km)}{283,15(K)}\right]^{\dfrac{16(\frac{kg}{kmol}) \cdot 9.81(\frac{m}{s^2})}{8.3145(\frac{kJ}{kmol \cdot K}) \cdot 30(K)}} = 86.077(bar)$$

Musterlösung Aufgabe 2: ≪Produktionspuffer-Tank≫



I. TEILAUFGABE A) ⇒ 4 PUNKTE

$\underline{\dot{V}_{ab} = ?}$

$\dot{m}_{ab} = \rho'(p_2) \cdot \dot{V}_{ab} \Rightarrow \dot{V}_{ab} = \frac{\dot{m}_{ab}}{\rho'(p_2)}$

$\dot{m}_{ab} = \dot{m}_1 = \dot{m} = \rho_1 \cdot \dot{V}_1 = \rho''(p_1) \cdot \dot{V}_1 = 3,369 \frac{kg}{m^3} \cdot 20 \frac{m^3}{h} = 67,38 \frac{kg}{h}$

$\Rightarrow \underline{\underline{\dot{V}_{ab}}} = \frac{67{,}38 \frac{kg}{h}}{625{,}7 \frac{kg}{m3}} = \underline{\underline{0,1077}} \frac{m^3}{h}$

$\underline{\dot{Q}_{12} = ?}$

$\underline{\underline{\dot{Q}_{12}}} = \dot{m} \cdot (h_2 - h_1) = \dot{m} \cdot (h'(p_2) - h''(p_1)) = 67,38 \frac{kg}{h} \cdot (-37,459 - 363,75) \frac{kJ}{kg} = \underline{\underline{-7,51 kW}}$

II. TEILAUFGABE B) ⇒ 4 PUNKTE

$\underline{\Delta t = ?}$ Zustand 2: vor Störung; Zustand 2*: nach Störung

$m_{ges,2} = \rho'(p_2) \cdot 0,6 \cdot V + \rho''(p_2) \cdot 0,4 \cdot V$

$= 625,7 \frac{kg}{m^3} \cdot 0,6 \cdot 10 m^3 + 1,7249 \frac{kg}{m^3} \cdot 0,4 \cdot 10 m^3 = 3761,1 kg$

$m_{ges,2*} = \rho'(p_2) \cdot 0,1 \cdot V + \rho''(p_2) \cdot 0,9 \cdot V = 625,7 \frac{kg}{m^3} \cdot 0,1 \cdot 10 m^3 + 1,7249 \frac{kg}{m^3} \cdot 0,9 \cdot 10 m^3 = 641,22 kg$

$\Delta m = m_{ges,2} - m_{ges,2*} = (3761,1 - 641,22) kg = 3119,88 kg$

$\Rightarrow \underline{\Delta t} = \frac{\Delta m}{\dot{m}} = \frac{3119{,}88 kg}{67{,}38 \frac{kg}{h}} = \underline{\underline{46,3 h}}$

III. TEILAUFGABE C) $\Rightarrow$ 4 PUNKTE

Füllstand a = ?

$$m_{ges,3} = m_{ges,2*} = \rho'(p_3) \cdot a \cdot V + \rho''(p_3) \cdot (1-a) \cdot V$$

$$\Rightarrow \underline{a} = \frac{m_{ges,3}/V - \rho''(p_3)}{\rho'(p_3) - \rho''(p_3)} = \frac{641{,}22kg/10m^3 - 4{,}5531\frac{kg}{m3}}{(595{,}3 - 4{,}5531)\frac{kg}{m3}} = \underline{\underline{10,08\%}}$$

$Q_{2*3} = ?$

1.HS:

$$Q_{2*3} + P_{2*3} = \Delta U_{2*3} = U_3 - U_{2*} = [m_{fl,3} \cdot u'(p_3) + m_{gas,3} \cdot u''(p_3)] - [m_{fl,2*} \cdot u'(p_2) + m_{gas,2*} \cdot u''(p_2)]$$

$$P_{2*3} = -\int p \cdot dV = 0$$

$$m_{fl,3} = \rho'(p_3) \cdot 0,1008 \cdot V = 600,28kg; \qquad m_{gas,3} = \rho''(p_3) \cdot 0,8992 \cdot V = 40,94kg$$

$$m_{fl,2*} = \rho'(p_2) \cdot 0,1 \cdot V = 625,7kg; \qquad m_{gas,2*} = \rho''(p_2) \cdot 0,9 \cdot V = 15,52kg$$

$$\Rightarrow \underline{\underline{Q_{2*3}}} = [600,28kg \cdot 33,257\frac{kJ}{kg} + 40,94kg \cdot 344,56\frac{kJ}{kg}] - [625,7kg \cdot (-37,55)\frac{kJ}{kg} + 15,52kg \cdot 299,86\frac{kJ}{kg}]$$

$$= \underline{\underline{52,91MJ}}$$

IV. TEILAUFGABE D) $\Rightarrow$ 2 PUNKTE

$p_4 = ?; \quad m_{max} = ?$

$$\underline{\underline{p_4}} = p_s(t_4 = -10°C) = \underline{\underline{0,015191MPa}}$$

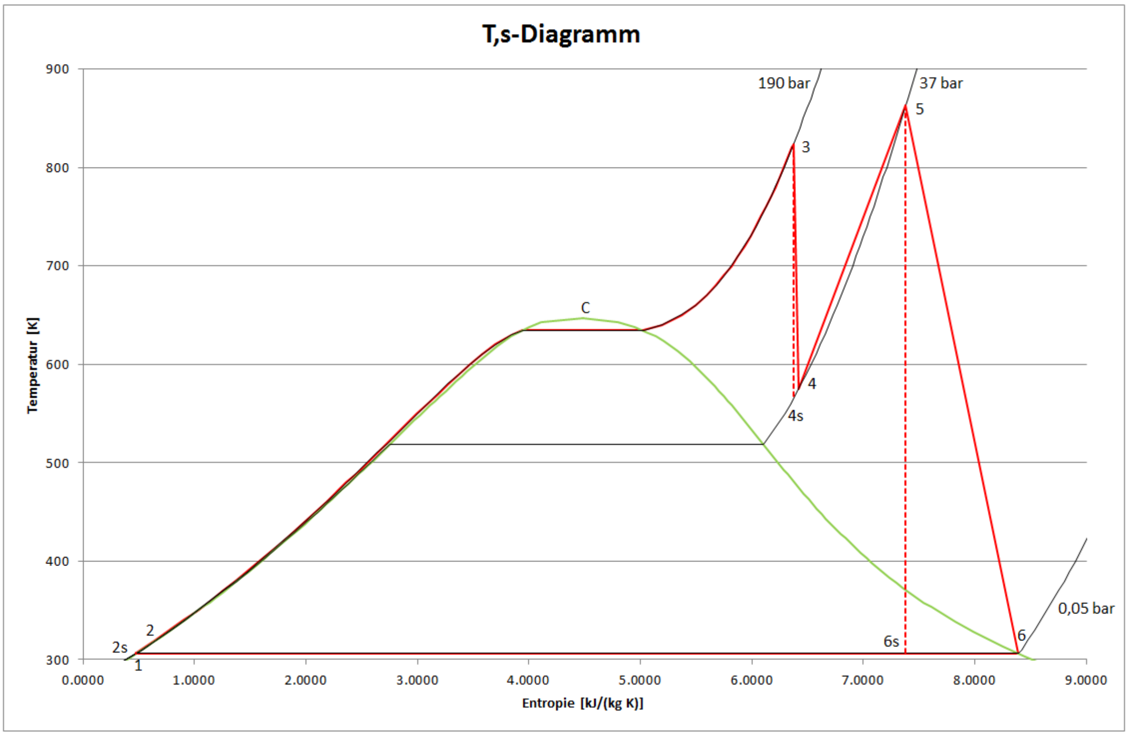
$$\underline{\underline{m_{max}}} = \rho'(p_4) \cdot a \cdot V + \rho''(p_4) \cdot (1-a) \cdot V$$

$$= \rho'(p_4) \cdot V; \qquad mit\ a = 1 \text{ (Füllstand 100 \%)}$$

$$= 654,22\frac{kg}{m^3} \cdot 10m^3 = \underline{\underline{6542,2kg}}$$

Musterlösung Aufgabe 3

I. TEILAUFGABE A)



II. TEILAUFGABE B)

$P_{56} = \dot{m}_w \cdot (h_6 - h_5)$

aus Tabelle abgelesen: $h_5(863.15K, 37bar) = 3654,35\ \frac{\mathrm{kJ}}{\mathrm{kg}}$

$\eta_{s,T} = \frac{w_{t56}}{w_{t56s}} = \frac{h_6 - h_5}{h_{6s} - h_5}$

$h_{6s} = h(s_{6s}, p_6) \qquad s_{6s} = s_5 = 7,3821\ \frac{kJ}{kg \cdot K}$

Dampfgehalt am Punkt 6s

$x_{6s} = \frac{s_{6s} - s_{6s'}}{s_{6s''} - s_{6s'}} = \frac{7,3821 - 0,4763}{8,3939 - 0,4763} = 0,8722$

$h_{6s} = h'_{6s} + x_{6s} \cdot (h_{6s''} - h_{6s'}) = 2251,132\ \frac{\mathrm{kJ}}{\mathrm{kg}}$

$h_6 = \eta_{s,T} \cdot (h_{6s} - h_5) + h_5 = 2321,293\ \frac{\mathrm{kJ}}{\mathrm{kg}}$

$\Rightarrow P_{56} = -79983,44\ \mathrm{kW}$

III. TEILAUFGABE C)

$\dot{Q}_{zu} = \dot{Q}_{23} + \dot{Q}_{45}$

$\dot{Q}_{23} = (h_3 - h_2) \cdot \dot{m}_w \qquad h_3(823.15\ K, 90\ bar) = 3407,31\ \frac{\mathrm{kJ}}{\mathrm{kg}} \quad \text{(abgelesen)}$

$\eta_{s,V} = \frac{w_{t12s}}{w_{t12}} = \frac{h_{2s} - h_1}{h_2 - h_1} = 0,8 \qquad h_{2s} = h(s_{2s},\ p_2) \qquad \mathrm{h}_1 = h'(p_1) = 137,77\ \frac{\mathrm{kJ}}{\mathrm{kg}}$

$s_{2s} = s_1 = 0,4763\ \frac{kJ}{kg \cdot K}$

$h_{2s} = \left(\frac{159,49 - 155,35}{0,4851 - 0,4716} \cdot (0,4763 - 0,4716) + 155,35\right) \frac{\mathrm{kJ}}{\mathrm{kg}} = 156,791\ \frac{\mathrm{kJ}}{\mathrm{kg}} \quad \text{(interpoliert)}$

$h_2 = 161,547\ \frac{\mathrm{kJ}}{\mathrm{kg}}$

$\Rightarrow \dot{Q}_{23} = 60 \frac{\mathrm{kg}}{\mathrm{s}} \cdot (3407,31 - 161,547) \frac{\mathrm{kJ}}{\mathrm{kg}} = 194,75\ \mathrm{MW}$

$\dot{Q}_{45} = \dot{m}_w \cdot (h_5 - h_4) \qquad h_5 = 3654,35\ \frac{\mathrm{kJ}}{\mathrm{kg}}$

$\eta_{s,T} = \frac{w_{t34}}{w_{t34s}} = \frac{h_4 - h_3}{h_{4s} - h_3} = 0,94 \qquad \mathrm{h}_{4s} = h(s_{4s},\ p_4) \qquad h_3 = 3407,31\ \frac{\mathrm{kJ}}{\mathrm{kg}}$

$s_{4s} = s_3 = 6,3732\ \frac{kJ}{kg \cdot K} \qquad h_{4s} = 2948,824 \frac{\mathrm{kJ}}{\mathrm{kg}} \quad \text{(interpoliert)}$

$\mathrm{h}_4 = 2976,333\ \frac{\mathrm{kJ}}{\mathrm{kg}}$

$\Rightarrow \dot{Q}_{45} = 60 \frac{\mathrm{kg}}{\mathrm{s}} \cdot (3654,35 - 2976,333) \frac{\mathrm{kJ}}{\mathrm{kg}} = 40,68\ \mathrm{MW}$

$\dot{Q}_{zu} = 40,68\ \mathrm{MW} + 194,75\ \mathrm{MW} = 235,43\ \mathrm{MW}$

$\eta_{th} = \frac{|P_{Nutz}|}{\dot{Q}_{zu}} \qquad P_{Nutz} = P_{12} + P_{34} + P_{56}$

$P_{12} = \dot{m}_w \cdot (h_2 - h_1) = 60 \frac{\mathrm{kg}}{\mathrm{s}} \cdot (161,547 - 137,77) \frac{\mathrm{kJ}}{\mathrm{kg}} = 1,427\ \mathrm{MW}$

$P_{34} = \dot{m}_w \cdot (h_4 - h_3) = 60 \frac{\mathrm{kg}}{\mathrm{s}} \cdot (2976,333 - 3407,31) \frac{\mathrm{kJ}}{\mathrm{kg}} = -25,857\ \mathrm{MW}$

$P_{56} = \dot{m}_w \cdot (h_6 - h_5) = 60 \frac{\mathrm{kg}}{\mathrm{s}} \cdot (2321,293 - 3654,35) \frac{\mathrm{kJ}}{\mathrm{kg}} = -79,98\ \mathrm{MW}$

$\Rightarrow P_{Nutz} = -104,415\ \mathrm{MW}$

$\eta_{th} = 0,4435$

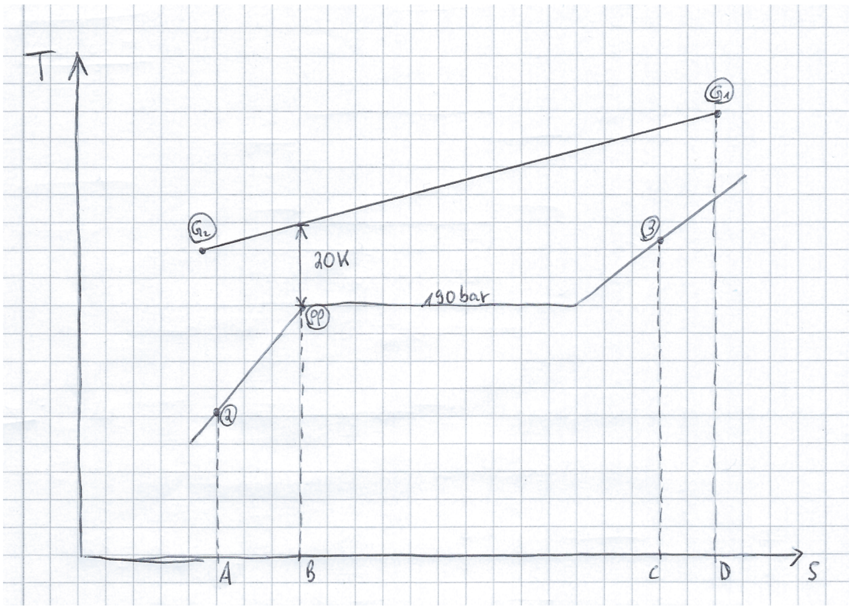
IV. TEILAUFGABE D)

$\Delta\dot{E}_V = \Delta\dot{E}_{Q23} + \Delta\dot{E}_{G1G2}$

$\Delta\dot{E}_{Q23} = \dot{m}_w \cdot ((h_3 - h_2) - T_a \cdot (s_3 - s_2))$ $\quad$ $s_3(823.15\ K,\ 190\ bar) = 6,3732\ \frac{kJ}{kg \cdot K}$

$s_2 = 0,491877\ \frac{kJ}{kg \cdot K}$ (interpoliert)

$\Rightarrow \Delta\dot{E}_{Q23} = 91,297$ MW



$\Delta\dot{E}_{G1G2} = \dot{m}_G \cdot ((h_{G2} - h_{G1}) - T_a \cdot (s_{G2} - s_{G1}))$

von G1 nach pp:

$\dot{Q}_{pp,3} = -\dot{Q}_{G1,B}$ $\quad$ $\dot{m}_w \cdot (h_3 - h_{pp}) = -\dot{m}_G \cdot (h_{GB} - h_{G1})$ $\quad$ $h_{pp} = h'(190\ bar) = 1776,89\ \frac{\mathrm{kJ}}{\mathrm{kg}}$

$\dot{m}_G = -\dfrac{\dot{m}_w \cdot (h_3 - h_{pp})}{c_p \cdot (T_{GB} - T_{G1})}$ $\quad$ $T_{GB} = 20K + T_{pp} = 20\ K + 634,621\ K = 654,621\ K$

$T_{G1} = 873,15\ K$ $\quad$ $h_3 = 3407,31\ \frac{\mathrm{kJ}}{\mathrm{kg}}$

$\Rightarrow \dot{m}_G = 389,264\ \frac{\mathrm{kg}}{\mathrm{s}}$

$\Delta\dot{E}_{G1G2} = \dot{m}_G \cdot \left[c_p \cdot (T_{G2} - T_{G1}) - T_a \cdot c_p \cdot \ln\left(\dfrac{T_{G2}}{T_{G1}}\right)\right]$

von GB nach G2:

$\dot{Q}_{pp,2} = -\dot{Q}_{GB,G2}$ $\quad$ $\Rightarrow \dot{m}_w \cdot (h_{pp} - h_{G2}) = -\dot{m}_G \cdot c_p \cdot (T_{G2} - T_{GB})$

$T_{G2} = 438,113\ K$

$\Rightarrow \Delta\dot{E}_{G1G2} = -104,246$ MW

$\Rightarrow \Delta\dot{E}_V = -12,949$ MW